

AP Calculus AB

Benchmark 1 • Chapters 1–3

Derivative (Chapter 1)

The **derivative** is one of the major concepts in Calculus.

1. Answer with a brief sentence: What does the derivative of a function tell us about the function?

2. Let $y = x^2 - 4x$. Estimate $y'(3)$ using a symmetric difference quotient.

3. Sketch the graph of $y = x^2 - 4x$. Explain the visual meaning of $y'(3)$ by using words and/or drawing on your sketch.

Definite Integral (Chapter 1)

The **definite integral** is another major concept in Calculus.

4. On your sketch from #3, shade the region of the graph represented by the definite integral from $x = 0$ to $x = 4$.

5. Use the trapezoid rule (with four trapezoids) to find the area under the curve of $y = x^2 - 4x$ from $x = 0$ to $x = 4$.

Limit (Chapter 2)

The **limit** is an important concept in Calculus because it helps us define the derivative.

6. Sketch the graph of some function $f(x)$ with

$$\lim_{x \rightarrow 3^-} f(x) = 4 \text{ and } \lim_{x \rightarrow 3^+} f(x) = 5.$$

7. Sketch the graph of some function $g(x)$ with

$$\lim_{x \rightarrow -\infty} g(x) = 2 \text{ and } \lim_{x \rightarrow \infty} g(x) = -\infty.$$

8. Show that $h(x) = \frac{3x^2 - 2x - 1}{x - 1}$ takes on an indeterminate form at $x = 1$.

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9. Evaluate $\lim_{x \rightarrow 1} \frac{3x^2 - 2x - 1}{x - 1}$ by removing the discontinuity.

Continuity (Chapter 2)

Certain results only apply when a function is **continuous**.

10. Is $y = \begin{cases} 3x - 2 & x < 4 \\ x^2 - 6 & x \geq 4 \end{cases}$ continuous at $x = 4$?

Justify your answer. (Hint: Three conditions...)

Intermediate Value Theorem (Chapter 2)

One important result of continuity is the **Intermediate Value Theorem** (IVT).

11. Paraphrase the conclusion (the “then” part) of the Intermediate Value Theorem in everyday language.

Concept of Derivative (Chapter 3)

In Chapter 3 we further developed our understanding of derivative.

12. State the algebraic definition of the derivative of a function.

13. Suppose the displacement of a particle is given by the function $d(t) = 3t^2$. Find the velocity and acceleration functions.

14. What is the difference between velocity and speed?

Techniques of Differentiation (Chapter 3)

Find the derivative of each function.

15. $y = x^2 + 6x + 5$

16. $y = 2^x$

17. $y = \cos x + \sin 3x$

18. $y = e^{-2x} + \ln 3x^5$

Equations of Tangent Lines (Chapter 3)

Write an equation for the line tangent to the given function at the indicated point.

19. $y = x^2 - 3x$ at $x = 5$

20. $y = \sin x + \cos x$ at $x = 0$