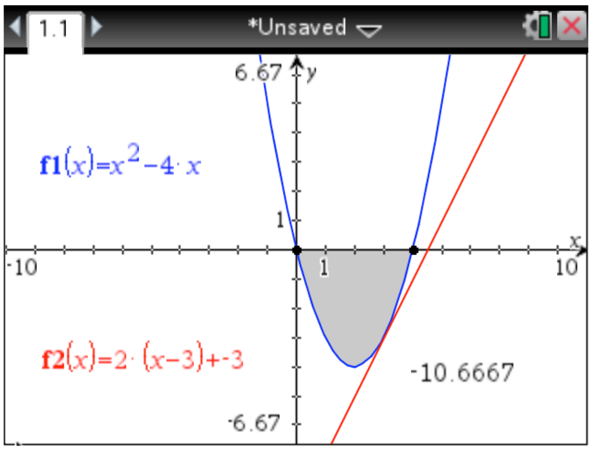
Derivative (Chapter 1)

1. The derivative of a function tells us the **instantaneous rate of change** of that function.

2. $y'(3) \approx \dfrac{y(3.1) - y(2.9)}{3.1 - 2.9} = 2$

3. The slope of the line tangent the the curve at $x = 3$ is $y'(3) = 2$.



Definite Integral (Chapter 1)

4. See above.

5. $T_4 = \frac{4-0}{4} \cdot \frac{1}{2} \cdot \left(y(0) + 2y(1) + 2y(2) + 2y(3) + y(4) \right) = -10$

Limit (Chapter 2)

6. Answers will vary.

7. Answers will vary.

8. $h(1) = \dfrac{3(1)^2 - 2(1) - 1}{1 - 1} = \dfrac{3 - 2 - 1}{0} = \dfrac{0}{0}$

9. $$\begin{aligned} &\lim_{x \to 1} \frac{3x^2 - 2x - 1}{x - 1} \\ &= \lim_{x \to 1} \frac{(x-1)(3x+1)}{(x-1)} \\ &= \lim_{x \to 1} (3x + 1) \\ &= 3 \cdot 1 + 1 \\ &= 4 \end{aligned}$$

Continuity (Chapter 2)

10. The function is continuous at $x = 4$ because the function exists there, the limit exists there, and the function and limit have the same value.

    In more formal terms, the function is continuous at $x = 4$ because…

    ✓ $y(4)$ exists: $y(4) = 4^2 - 6 = 10$

    ✓ $\lim_{x \to 4^-} y = 3 \cdot 4 - 2 = 10$

    $\lim_{x \to 4^+} y = 4^2 - 6 = 10$

    That is, $\lim_{x \to 4^-} y = \lim_{x \to 4^+} y$

    ✓ $\lim_{x \to 4^{-1}} y = y(4)$

Intermediate Value Theorem (Chapter 2)

11. If a function $f(x)$ is continuous on some closed interval $[a, b]$, and there is some y value between $f(a)$ and $f(b)$, then there is guaranteed to be some $x = c$ in the open interval (a, b) such that $f(c) = y$.

Concept of Derivative (Chapter 3)

12. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

13. $v(t) = d'(t) = 6t$

    $a(t) = v'(t) = d''(t) = 6$

14. $speed = |velocity|$

Updated: Oct 1, 2012

Techniques of Differentiation (Chapter 3)

15. $y' = 2x + 6$

16. $y' = 2^x \cdot \ln 2$

17. $y' = -\sin x + 3\cos 3x$

18. $y' = e^{-2x} \cdot -2 + \dfrac{1}{3x^5} \cdot 15x^4$

$= -2e^{-2x} + \dfrac{5}{x}$

Equations of Tangent Lines (Chapter 3)

19. Since $y = x^2 - 3x$, $y' = 2x - 3$.

Note that $y(5) = 10$ and $y'(5) = 7$.

Thus, an equation of the line tangent to $y = x^2 - 3x$ at $x = 5$ is given by:

$$y = 7(x - 5) + 10$$

or

$$y = 7x - 25$$

20. Since $y = \sin x + \cos x$, $y' = \cos x - \sin x$.

Note that $y(0) = 1$ and $y'(0) = 1$.

Thus, an equation of the line tangent to $y = \sin x + \cos x$ at $x = 0$ is given by:

$$y = 1(x - 0) + 1$$

or

$$y = x + 1$$